**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. **0.2676**
4. 0.5
5. 0.6987

Ans: We have a normal distribution with *μ* = 45 and *σ* = 8.0.

Let X be the amount of time it takes to complete the repair on a customer's car. To finish in one hour you must have X ≤ 50 so the question is to find P(X > 50).

P(X > 50) = 1 -P(X ≤ 50).

Z = (X-μ)/б

Z = (X - 45)/8

Thus the question can be answered by using the normal table to find P(X ≤ 50) = P(Z ≤ (50 -45)/8.0) = P(Z ≤ 0.625)=73.4%

Probability that the service manager will not meet his demand will be = 100-73.4 =26.6% or 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.

**Given:** Mean *μ* = 38 and Standard deviation *σ* =6

1. More employees at the processing center are older than 44 than between 38 and 44.

Ans: Probability of employees greater than age of 44= P(X>44)

P(X > 44) = 1 -P(X ≤ 44)

Z = (X-μ)/б

Z= (X -38)/6

Thus the question can be answered by using the normal table to find

P(X ≤ 44) = P(Z ≤ (44 -38)/6) = P(Z ≤ 1)=84.1345%

Probability that the employee will be greater than age of 44

= 100-84.1345 = 15.86%

Thus the question can be answered by using the normal table to find

P(X ≤ 44) = P(Z ≤ (44 -38)/6) = P(Z ≤ 1)=84.1345%

Probability that the employee will be greater than age of 44

= 100-84.1345 = 15.86% ≈ 63 out of 400

Z score for 38 = (38 - 38)/6 = 0 => 50%

Hence People between 38 & 44 age = 84.13 - 50 = 34.13 % ≈ 137 out of 400

Hence More employees at the processing center are older than 44 than between 38 and 44. is **FALSE**

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans: Probability of employees less than age of 30 = P(X<30)

Z= (X -38)/6

= (30 - 38)/6 = -1.33

P(X ≤ 30) = P(Z ≤ (30 -38)/6) = P(Z ≤ -1.333)=9.12%9.15 % ≈ 36 out of 400

Hence A training program for employees under the age of 30 at the center would be expected to attract about 36 employees - **TRUE**

1. If *X1* ~ *N* (μ, σ2) and *X*2 ~ *N* (μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Ans:** As we know that if *X1* ∼ N (µ1, σ12), and *X2* ∼ N (µ2, σ22) are two independent random variables, then *X1* + *X2* ∼ N (µ1 + µ2, σ12+ σ22), and *X1* - *X2* ∼ N (µ1 - µ2, σ12 - σ22).

Similarly, if Z = a *X1* + b *X2*, where *X1* and *X2* are as defined above, i.e., Z is linear combination of *X1* and *X2*, then Z ∼ N (aµ1 + bµ2, a^2σ1^2 + b^2σ2^2).

Therefore, in the question

2 *X1* ~ N (2 μ,4 σ2) and

*X1* + *X2* ~ N (µ + µ, σ2+ σ2) ~ N (2 µ, 2 σ2)

2 *X1* -( *X1* + *X2*) = N (4µ, σ2)

1. Let X ~ N (100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

**Ans:** Since we need to find out the values of a and b, which are symmetric about the mean, such that the probability of random variable taking a value between them is 0.99. We have to work out in reverse order.

The Probability of getting value between a and b should be 0.99.

So, the Probability of going wrong, or the Probability outside the a and b area is 0.01 (i.e., 1-0.99).

The Probability towards left from a = -0.005 (i.e., 0.01/2).

The Probability towards right from b = +0.005 (i.e., 0.01/2).

So, since we have the probabilities of a and b, we need to calculate X, the random variable at a and b which has got these probabilities.

By finding the Standard Normal Variable Z (Z Value), we can calculate the X values.

Z= (X- μ) / σ

For Probability 0.005 the Z Value is -2.57 (from Z Table).

Z \* σ + μ = X

Z (-0.005) \*20 + 100 = -(-2.57) \* 20 + 100 = 151.4

Z (+0.005) \*20 + 100 = (-2.57) \* 20 + 100 = 48.6

So, option D is correct.

1. 90.5, 105.9
2. 80.2, 119.8
3. 22, 78
4. **48.5, 151.5**
5. 90.1, 109.9
6. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
7. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans: Rupee ranges in between (99.00810347848784, 980.9918965215122) Millions, 95% of the time for the Annual Profit of the Company.

By using stats.norm.interval(0.95,540,225) function

1. Specify the 5th percentile of profit (in Rupees) for the company

Ans: 5th percentile of profit (in Million Rupees) is 170.0

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans: The Division Profit2 ~ N(7, 42) has a larger probability of making a loss in a given year.